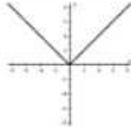
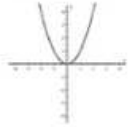
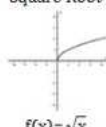

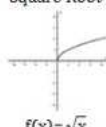

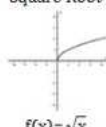



**Integrated Math 3- End of Course Final Review Packet**

**Part 1: Functions**

**Transformations of Parent Functions**

<p><b>Absolute Value</b></p>  <p><math>f(x) =  x </math></p>	<p><b>Quadratic</b></p>  <p><math>f(x) = x^2</math></p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Transformation</th> <th>Appearance in Function</th> </tr> </thead> <tbody> <tr> <td>Vertical Translation</td> <td><math>f(x) \rightarrow f(x) + d</math> →</td> </tr> <tr> <td>Horizontal Translation</td> <td><math>f(x) \rightarrow f(x - c)</math> →</td> </tr> <tr> <td>Vertical Stretch/Compression</td> <td><math>f(x) \rightarrow af(x)</math></td> </tr> <tr> <td>Reflection in x-axis</td> <td><math>f(x) \rightarrow -f(x)</math></td> </tr> <tr> <td>Horizontal Stretch/Compression</td> <td><math>f(x) \rightarrow f(kx)</math></td> </tr> <tr> <td>Reflection in y-axis</td> <td><math>f(x) \rightarrow f(-x)</math></td> </tr> </tbody> </table>	Transformation	Appearance in Function	Vertical Translation	$f(x) \rightarrow f(x) + d$ →	Horizontal Translation	$f(x) \rightarrow f(x - c)$ →	Vertical Stretch/Compression	$f(x) \rightarrow af(x)$	Reflection in x-axis	$f(x) \rightarrow -f(x)$	Horizontal Stretch/Compression	$f(x) \rightarrow f(kx)$	Reflection in y-axis	$f(x) \rightarrow f(-x)$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: center;"> <p><b>Square Root</b></p>  <p><math>f(x) = \sqrt{x}</math></p> </td> <td style="width: 50%; text-align: center;"> <p><b>Cubic</b></p>  <p><math>f(x) = x^3</math></p> </td> </tr> </table>	<p><b>Square Root</b></p>  <p><math>f(x) = \sqrt{x}</math></p>	<p><b>Cubic</b></p>  <p><math>f(x) = x^3</math></p>
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#1-4. Describe the transformations from the graph of  $f(x)$  to the graph of  $g(x)$ .

1.  $f(x) = x^2, g(x) = -(x + 1)^2 - 3$
2.  $f(x) = |x|, g(x) = |x - 4| + 6$
3.  $f(x) = \sqrt{x}, g(x) = -\sqrt{x + 3} - 1$
4.  $f(x) = x^3, g(x) = (-x)^3 + 2$

**Domain and Range**

Parent Function	Domain (x-values)	Range (y-values)
$y = x^2$	$x = \text{All Real Numbers}$	$y \geq 0$
$y = \sqrt{x}$	$x \geq 0$	$y \geq 0$
$y =  x $	$x = \text{All Real Numbers}$	$y \geq 0$
$y = \frac{1}{x}$	$x = \text{All Real Numbers, except } x \neq 0$	$y > 0$

5. Find the domain of  $f(x) = \sqrt{x - 1} + 6$ .
6. Find the domain of  $f(x) = -(x + 4)^2 - 3$ .
7. Find the range of  $f(x) = (x + 2)^2 + 3$
8. Find the range of  $f(x) = |x + 1| - 2$

**Composing Functions**

9. Given  $f(x) = 2x - 1$  and  $g(x) = \frac{x+3}{2}$ , find  $f(g(x))$ .
10. Given  $f(x) = x^2$  and  $g(x) = x + 4$ , find  $f(g(x))$ .

11. Given  $f(x) = 3x + 1$  and  $g(x) = 2x - 4$ , find  $g(f(x))$ .

12. Given  $f(x) = \frac{x+6}{3}$  and  $g(x) = 3x + 1$ , find  $g(f(x))$ .

### Inverse Functions

Find the inverse of  $f(x) = 3x - 2$

$$f(x) = 3x - 2$$

$$y = 3x - 2 \quad \longrightarrow \quad x = 3y - 2$$

$$x + 2 = 3y$$

$$\frac{x+2}{3} = y$$

$$f^{-1}(x) = \frac{x+2}{3}$$

Step #1: Switch x and y

Step #2: Isolate y.

**If  $f(x)$  and  $g(x)$  are inverse functions, then  $f(g(x)) = x$ .**

13. Find the inverse of  $f(x) = 3x^2 + 1$ .

14. Find the inverse of  $f(x) = \sqrt[3]{x+3} - 2$ .

15. Are  $f(x)$  and  $g(x)$  inverse functions?

$$f(x) = 2x - 1, \quad g(x) = \frac{x}{2} + 1$$

16. Are  $f(x)$  and  $g(x)$  inverse functions?

$$f(x) = 5x + 1, \quad g(x) = \frac{x-1}{5}$$

### Even & Odd Functions

<b>Even Functions</b>	<b>Odd Functions</b>
$f(-x) = f(x)$	$f(-x) = -f(x)$

#17-20. Determine if the functions are even or odd, or neither.

17.  $f(x) = 3x^4 - 2x^2 + 8$

18.  $f(x) = 2x^3 - 4x$

19.  $f(x) = x^2 - 2x + 1$

20.  $f(x) = 4x^2 + 5$

### Rate of Change

The rate of change of a function from  $x = a$  to  $x = b$  is  $\frac{f(a)-f(b)}{a-b}$ .

21. Find the rate of change of  $f(x) = x^2 + 2$  from  $x = -1$  to  $x = 3$ .

22. Find the rate of change of  $f(x) = \sqrt{3x - 2}$  from  $x = 1$  to  $x = 6$ .

23. Find the rate of change of  $f(x) = \sqrt{1 - 2x}$  from  $x = -12$  to  $x = -4$ .

24. Find the rate of change of  $f(x) = 2x - 3$  from  $x = -2$  to  $x = 2$ .

### Part 2: Logarithmic and Exponential Functions

#### Evaluating Logarithms

Rewrite as an Exponential Function

$$\log_b y = x \rightarrow b^x = y$$

Use Change of Base Formula (Calculator)

$$\log_b y = \frac{\log y}{\log b}$$

#25-28. Evaluate the Logarithms.

25.  $\log_3 27$

26.  $\log_5 25$

27.  $\log_4 16$

28.  $\log_2 16$

#### Expanding Logarithms

1.  $\log_b mn = \log_b m + \log_b n$

2.  $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

3.  $\log_b m^r = r \log_b m$

29. Simplify and Expand  $\log_2 \frac{4x^2y}{2x}$ .

30. Simplify and Expand  $\log_3 \frac{x^5y^2}{x^3y^3}$ .

31. Expand  $\log 24$  in two different ways.

32. Expand  $\log 36$  in two different ways.

**Solving Logarithms & Exponential Equations**

33.  $4^{x-2} = 16^{x+1}$

34.  $27^{2x} = 3^{x+10}$

35.  $\log_3(x + 2) - \log_3 5 = 3$

36.  $\log_6 2x + \log_6 3 = 4$

37.  $\log_2(7 - 3x) = 4$

38.  $\log_5(2x + 1) = 2$

39.  $4e^{3x} = 12$

40.  $-6e^{2x} = -36$

41.  $e^{4x} - 2 = 6$

42.  $e^{2x} = 1.5$

**Exponential Growth vs. Exponential Decay**

$$f(x) = a(b)^x$$

Exponential Growth:  $b > 1$ .

Exponential Decay:  $0 < b < 1$ .

#43-46. Classify the following as exponential growth or exponential decay.

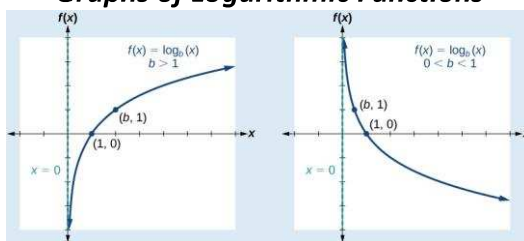
43.  $f(x) = 12(1.25)^x$

44.  $f(x) = 4\left(\frac{5}{4}\right)^x$

45.  $f(x) = 15(0.5)^x$

46.  $f(x) = 2.3^x$

### Graphs of Logarithmic Functions



#47-50. Find the x-intercept and Vertical Asymptote for the graph of each function.

47.  $f(x) = \log_2(x - 3)$

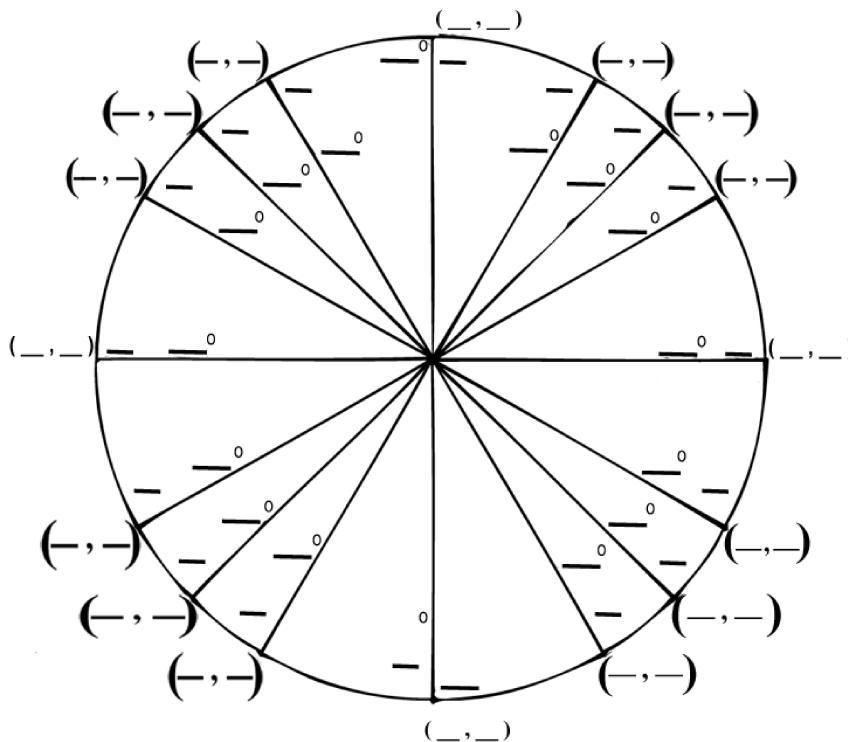
48.  $f(x) = \log_5 x - 2$

49.  $f(x) = \log_3(x + 2)$

50.  $f(x) = \log_2 x + 1$

### Part 3: Trigonometry

Fill in the Unit Circle



### Evaluating Trigonometric Functions

51.  $\cos\left(\frac{\pi}{3}\right)$

52.  $\sin(120^\circ)$

53.  $\cos\left(\frac{7\pi}{6}\right)$

54.  $\sin\left(\frac{5\pi}{4}\right)$

55.  $\sin \pi$

56.  $\cos 0$

57.  $\cos(-45^\circ)$

58.  $\sin(-60^\circ)$

59.  $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ . Find another value for  $\theta$  where  $\sin \theta = \frac{1}{2}$ .

Name two angles that have the opposite value of  $\sin\left(\frac{\pi}{6}\right)$ .

60.  $\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$ . Find another value for  $\theta$  where  $\cos \theta = \frac{-\sqrt{2}}{2}$ .

Name two angles that have the opposite value of  $\cos\left(\frac{3\pi}{4}\right)$ .

### Converting Between Radians and Degrees

Degrees to Radians

$$x^\circ \cdot \frac{\pi}{180^\circ}$$

Radians to Degrees

$$x \cdot \frac{180^\circ}{\pi}$$

#61-64. Convert the angle measure from radians to degrees.

61.  $\frac{5\pi}{4}$

62.  $\frac{\pi}{2}$

63.  $\frac{11\pi}{6}$

64.  $\frac{9\pi}{4}$

#65-68. Convert the angle measure from degrees to radians.

65.  $120^\circ$

66.  $480^\circ$

67.  $225^\circ$

68.  $300^\circ$

### Graphing Trigonometric Functions

$$y = A \sin B(\theta - C) + D$$

A	B	C (Inside Parentheses)	D
<p><b>A= Amplitude</b> Tells you the <b>Maximums and Minimums</b> If A is negative, there is a reflection over the x-axis (Maximums and Minimums change places- First "M" is a minimum)</p>	<p><b>B= Frequency</b> Used to find the Period <b>Period = <math>\frac{2\pi}{B}</math></b></p>	<p><b>Phase Shift = <math>\frac{C}{B}</math></b> If <math>\frac{C}{B}</math> is <b>positive</b>, the graph shifts <b>left</b>. If <math>\frac{C}{B}</math> is <b>negative</b>, the graph shifts <b>right</b>. <math>\frac{\pi}{2}</math> is 2 units, <math>\frac{\pi}{4}</math> is 1 unit</p>	<p><b>Vertical Shift = D</b> If D is positive, the graph shifts up. If D is negative, the graph shifts down.</p>

#69-72. Identify the Amplitude, Period, and Shifts for the graphs below.

69.  $f(x) = 3\sin(\pi x)$

70.  $f(x) = -\cos 2(x - 1)$

A= \_\_\_\_\_ P= \_\_\_\_\_ PS= \_\_\_\_\_ VS= \_\_\_\_\_

A= \_\_\_\_\_ P= \_\_\_\_\_ PS= \_\_\_\_\_ VS= \_\_\_\_\_

71.  $k(t) = 20 + 2 \cos(\pi t)$

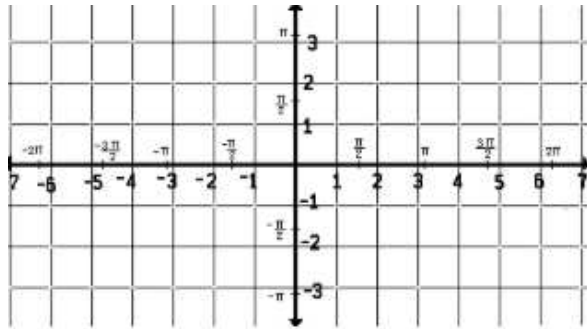
72.  $h(t) = 2 \sin(\pi + 2) - 3$

A= \_\_\_\_\_ P= \_\_\_\_\_ PS= \_\_\_\_\_ VS= \_\_\_\_\_

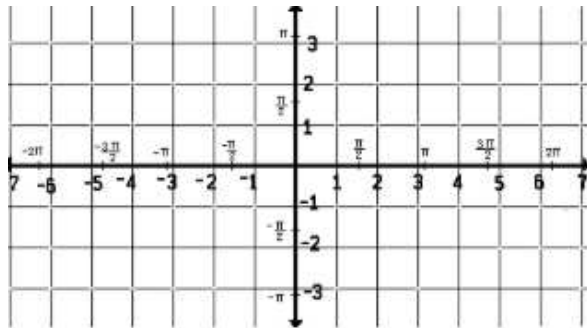
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#73-76. Graph the trigonometric functions.

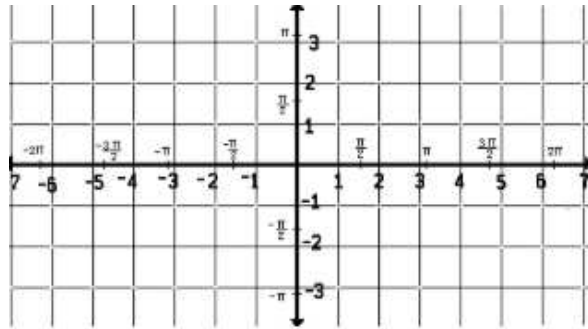
73.  $f(x) = -2\sin(2x)$



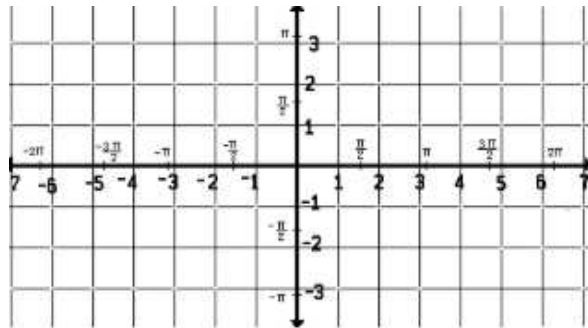
75.  $f(x) = \cos(x) + 2$



74.  $f(x) = 3\cos(\pi x)$



76.  $f(x) = \tan x$

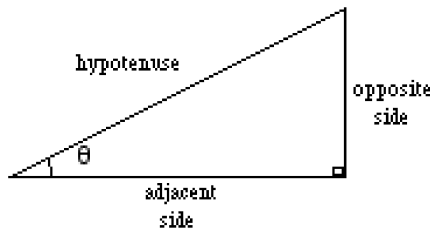


**Using Right Triangles to Find Trigonometric Values**

SOH  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

CAH  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

TOA  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$



Pythagorean Theorem:  
 $a^2 + b^2 = c^2$

77. If  $\tan B = \frac{3}{4}$  and  $\sin B = \frac{3}{5}$ , find  $\cos B$ .

78. If  $\tan \theta = \frac{5}{12}$ , find  $\sin \theta$ .

79. If  $\cos \theta = \frac{3}{5}$  and  $\sin \theta$  is positive, find  $\tan \theta$ .

80. If  $\sin \theta = \frac{8}{17}$  and  $\cos \theta$  is negative, find  $\tan \theta$ .

### Types of Statistical Studies

#81-83. Classify the study as a survey, an observational study, or an experiment.

81. Apple gathers 100 college students and monitors their reactions while completing an assignment on the newest MacBook Air.

82. A store emails a questionnaire to 1,000 randomly chosen online customers.

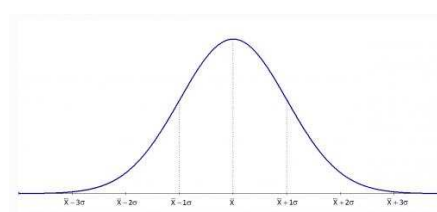
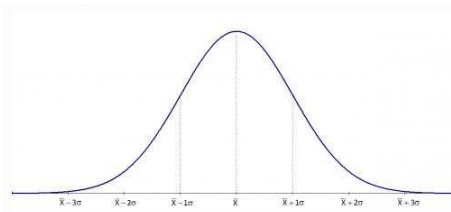
83. In-n-out is thinking about adding more relish to their spread. They put the new spread on half of their orders for one day, and then ask the customers their opinion.

### Interpreting Survey Results.

84. When surveying students on their Snapchat Scores, 2 out of 25 students reported not having a Snapchat. What proportion of the sample population did have a Snapchat?

85. 150 out of 250 students reported spending more than \$300 on prom. What proportion of the population of prom attendees spent more than \$300?

### Normal Distribution



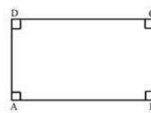
86. The number of years that people spend on their college education is normally distributed with a mean of 4 and a standard deviation of 1. What percentage of the population attends college between 4 and 5 years?

87. Movie prices at a theater are normally distributed with a mean of \$10.50 and a standard deviation of \$0.50. What price range represents the middle 68% of movie prices?

### Part 5: Solids

88. Mrs. Pots wants to mail her son Chip some of his favorite soup. She has 12 cans and plans to pack them in a  $3 \times 4 \times 1$  formation. Each can has a diameter of 3 inches and is 5 inches tall. If she wants to put 1 inch of packing between each can, on all sides, and on the top and the bottom, what are the dimensions of the box she would need?

89. What solid would be formed if you rotated Rectangle ABCD around side CD?



90. What solid would be formed if you rotated Triangle ABC around side BC?

